

OPTIMIZATION OF VALUABLE MINERALS (Mg^{2+} & K^+) RECOVERY FROM SEAWATER USING RESPONSE SURFACE METHODOLOGY (RSM)

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Abstract

The extraction of metallic ions from seawater brine within a circular economy framework presents economic promise yet is hampered by concomitant ionic interference manifested during the precipitation of salts. The present investigation sought to characterize and subsequently to refine the co-precipitation steps for magnesium and potassium ions such that they yield suitable precursors for the synthesis of struvite fertilizer. The experimental work utilized a Response Surface Methodology built upon a Face-Centered Central Composite Design, interrogating the influence of the molar ratio of sodium hydrogen phosphate to calcium chloride and the duration of reaction on the simultaneous recovery of Mg^{2+} and K^+ . Results from the Analysis of Variance demonstrated that the principal effects of the aforementioned ratio, reaction duration, and their interaction terms were statistically significant for both target recoveries. By iterative optimization, the experimental framework predicted optimal operational parameters of sodium hydrogen phosphate to calcium chloride ratio of 0.401 and a reaction interval of 11.608 minutes, corresponding to a composite desirability statistic of 0.908 and estimated recoveries of 93.979 % and 97.300 % for magnesium and potassium ions, respectively. The data therefore substantiate the conclusion that Response Surface Methodology provides a systematic and analytically tractable conduit for the resolution of equilibrated brine mixtures, delineating a refined process envelope that promotes the extraction and economic valorization of essential mineral fertilizers from industrial saline streams.

Keywords: Co-precipitation, Response Surface Methodology (RSM), Seawater

Abstrak

Ekstraksi ion-ion logam dari air garam (*brine*) air laut dalam kerangka ekonomi sirkular menyajikan potensi ekonomi, namun terhambat oleh interferensi ionik penyerta yang termanifestasi selama proses presipitasi garam. Penelitian ini bertujuan untuk mengkarakterisasi dan selanjutnya menyempurnakan langkah-langkah ko-presipitasi ion magnesium dan kalium sehingga menghasilkan prekursor yang sesuai untuk sintesis pupuk struvite. Pekerjaan eksperimental ini memanfaatkan Metodologi Permukaan Respon (*Response Surface Methodology*) yang dibangun berdasarkan Rancangan Komposit Pusat Berpusat Muka (*Face-Centered Central Composite Design*), untuk menguji pengaruh rasio molar natrium hidrogen fosfat terhadap kalsium klorida dan durasi reaksi terhadap perolehan kembali Mg^{2+} dan K^+ secara simultan. Hasil dari Analisis Varians (ANOVA) menunjukkan bahwa pengaruh utama dari rasio, durasi reaksi, dan suku interaksi keduanya adalah signifikan secara statistik untuk kedua target perolehan kembali. Melalui optimasi berulang, kerangka kerja eksperimental ini memprediksi parameter operasional optimal pada rasio natrium hidrogen fosfat terhadap kalsium klorida sebesar 0,401 dan interval reaksi 11,608 menit, yang bersesuaian dengan statistik *desirability* komposit sebesar 0,908 serta estimasi perolehan kembali masing-masing sebesar 93,979% untuk ion magnesium dan 97,300% untuk ion kalium. Oleh karena itu, data ini memperkuat kesimpulan bahwa Metodologi Permukaan Respon menyediakan jalur yang sistematis dan dapat ditangani secara analitis untuk resolusi campuran air garam setimbang, serta menggambarkan rentang proses yang telah disempurnakan yang mendorong ekstraksi dan valorisasi ekonomi pupuk mineral esensial dari aliran salin industri.

Keywords: Air Laut, Ko-presipitasi, Metodologi Permukaan Respon (RSM)

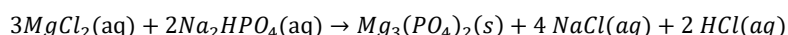
1. Introduction

Recovery of economically important minerals from seawater and atmospheric brines offers a pivotal lever for the ocean economy, simultaneously mitigating global resource shortfalls and converting waste from seawater desalination and industrial salinization into economically advantaged streams. In both sectors, ions such as Mg^{2+} , K^+ , and Ca^{2+} are routinely classified as impurities whose reductive removal is mandatory for the attainment of metric-grade salinity levels. Conceived through a circular economy lens, these ions in reality represent recyclable biovinyls that are standardised for plant nutrition. The present investigation is consequently oriented towards the elaboration of a cascade scheme that extracts these cationic substrates for transformation into precursor-grade raw materials that subsequently facilitate the economically valorized subsynthesis of slow-release struvite entrapped fertilizers.(Zhang et al., 2021)(Le Corre et al., 2009)

Realization of such a selective recovery pathway via chemical precipitation remains, at the thermodynamic and kinetic levels, an ergonomic enterprise. The operating discipline is invariably informed by fundamental laws of solubility thermodynamics, solution activity equilibria and customised, stochastic-fined, reaction kinetics anchored in a multicomponents science sphere. More narrowly, the simultaneous cation(ns)-coprecipitation of both Mg^{2+} and K^+ , instigated by a controlled addition of phosphoric-alkali nano-chemical co-eds, degrades into a multiplicity of concurrent, elaborated crystal hybrid sustainable pathways. The yielded hybrid network, therefore, dissolves into a simultaneous, multi-lagged individual, quick heterogeneous reaction sequence that desensitizes classical optimization. Laged, structural phase traits at a combined crystal level, vanish and network dynamics reverse, breaked that dependence abandons the feasible equilibrium without systematic, bespoke, multidimensional insights.(Loganathan et al., 2017)(Wei et al., 2025)

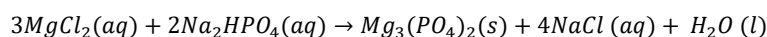
Given the inherent chemical complexity of a saline matrix, the primary objective of this research is to analyze and optimize the co-precipitation of Mg^{2+} and K^+ ions. This is accomplished through a chemical treatment process involving disodium phosphate (Na_2HPO_4), calcium chloride ($CaCl_2$), and sodium hydroxide ($NaOH$). (Pujiastuti et al., n.d.)The necessity for a systematic approach arises from the multifaceted and interacting roles of these reagents. The core precipitation is driven by Na_2HPO_4 , which provides the essential phosphate anion (PO_4^{3-}). However, its active form, the orthophosphate ion, only becomes sufficiently available at high alkalinity. Therefore, $NaOH$ is added to elevate the pH, shifting the phosphate equilibrium to drive the reaction forward. This action is not without consequence, as it simultaneously introduces a critical competing reaction: the precipitation of magnesium hydroxide ($Mg(OH)_2$), which must be carefully managed. Furthermore, the process must account for interfering ions, notably sulfate (SO_4^{2-}), whose co-precipitation is mitigated by adding $CaCl_2$ to form sparingly soluble calcium sulfate ($CaSO_4$). Due to these complex synergies, competing side-reactions, and purification steps, a method capable of navigating the interactions between process variables is required. Consequently, Response Surface Methodology (RSM) is employed to systematically optimize the conditions for maximizing the selective recovery of the target minerals (Zaid et al., 2022)

The chemical process begins with the introduction of the phosphate source into the magnesium-rich solution. Under the alkaline conditions established by $NaOH$, the phosphate ions can react with magnesium, leading to the formation of magnesium phosphate ($Mg_3(PO_4)_2$) as a potential solid intermediate or competing product. This initial precipitation step can be represented as:

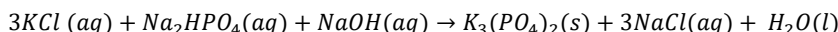


Concurrently, the system contains a significant concentration of potassium ions. While a simple precipitation of potassium phosphate (K_3PO_4) might be hypothesized, this pathway is chemically unfavorable due to the compound's high aqueous solubility. Instead, the process is engineered to favor the incorporation of potassium ions into a more stable, ternary (three-component) crystalline structure along with magnesium and phosphate. This prevents the formation of separate, simple phosphate salts and instead promotes the co-precipitation of the target mineral.

Therefore, the principal objective is to guide these individual ion interactions towards the formation of a single, less soluble compound: Magnesium Potassium Phosphate Hexahydrate ($MgKPO_4 \cdot 6H_2O$), known as K-struvite. The following step reaction, which represents the successful co-precipitation of the target product, can be summarized as follows:

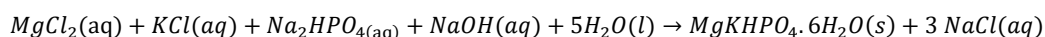


Also, under alkaline conditions (controlled with NaOH) K⁺ ions could, in principle, combine with phosphate ions to yield potassium phosphate (K₃PO₄) as given by the reaction:



The solubility of these compounds is determined by the pH and concentration of the reactants. Once the compounds are formed, they can be easily separated from the liquids by filtration or gravity.

K₃(PO₄)₂ dissolves easily in water, although, like all salts, its solubility is influenced by temperature, pH, and the concentration of other ions. More often, one encounters the precipitate of magnesium potassium phosphate, which results from the interaction of magnesium chloride, potassium chloride, and disodium hydrogen phosphate in the presence of sodium hydroxide, provided the mixture is kept in strongly alkaline conditions. This reaction can be summarised as follows:



Response Surface Methodology (RSM) is an empirical modeling technique concerned with exploring the functional relationships between several explanatory (independent) variables and one or more response (dependent) variables. (MOHAMMED et al., 2025) Its primary objective is to optimize this response by determining the set of input variable conditions that results in a maximum, minimum, or target value. For instance, this could involve maximizing valuable mineral recovery efficiency, minimizing operational cost, or achieving a specific product purity.

The superiority of Response Surface Methodology (RSM) over traditional experimental strategies resides principally in its integrated capacity for rapid, multidimensional analysis. By executing a carefully structured fraction of the full factorial space, RSM simultaneously addresses several experimental objectives. The technique quantifies the individual contributions of multiple independent variables, estimates interaction terms that may exhibit complementary or inhibitory behavior, and derives a compact, predictive polynomial representation of the system. This representation subsequently serves to extrapolate guidelines for optimum operating procedures that maximize a prescribed response metric. (Raziani & Mohammadidoust, 2020). The resulting operational profile guarantees that experimental data density remains high while the absolute experimental burden remains low, thus optimizing, time, materials, and expenditures. (MOHAMMED et al., 2025)

Building an accurate response surface model necessitates the application of carefully constructed experimental designs that generate an adequate volume of data for the reliable estimation of the coefficients in a polynomial formulation. Two of the most widely adopted designs for optimization purposes are the Central Composite Design (CCD) and the Box-Behnken Design (BBD). The CCD is characterised by a high degree of efficiency and adaptability, making it particularly appropriate for the subsequent fitting of a second-order (quadratic) model. Its physical learning structure comprises three components: a full factorial array occupying the vertices of the experimental region of interest, a set of axial (or star) points that extend symmetrically upon the orthogonal axes of the design, and an assembly of replicated central points employed to enhance the robustness of the resulting model by controlling for curvature and improved precision. (Raziani & Mohammadidoust, 2020)

The Box-Behnken Design (BBD) presents an efficient three-level construction for estimating quadratic response surfaces while deliberately omitting runs at the vertex (corner) levels of the experimental space. This characteristic is valuable when experimental realizations at these extremities either introduce operational hazards, incur excessive cost, or violate system constraints, thereby permitting the experimenter to conduct an economical, hazard-minimized, and technically feasible study over the central and midrange levels of the factors of interest. The design preserves rotatability and usually requires fewer runs than a full central-composite, thereby Streamlining the optimization. (Alam et al., 2022)

This study employs Response Surface Methodology (RSM) to maximize the selective harvesting of essential mineral species—specifically magnesium and potassium ions—suitable for the synthesis of struvite from marine brines, thereby enabling effective feedstock preparation in salt crystallization refinement. RSM systematically modulates the molar ratio of sodium hydrogen phosphate (Na_2HPO_4) to calcium chloride (CaCl_2), while simultaneously governing the solution pH via sodium hydroxide (NaOH) to foster in situ hydroxide co-precipitation. Consequently, the coupled adjustments of these two parameters engender a controllable reaction pathway, facilitating the co-precipitation of struvite and other contaminating species to enhance product recovery and, reciprocally, mineral yield. The overarching aim remains to retain calcium and magnesium in a form compatible with subsequent struvite synthesis, thereby providing a dual economic and environmental rationale for the valorization of seawater as a mineral feedstock.

2. Methodology

2.1. Material

The seawater used in this study was sourced from the coast of Sidoarjo, East Java, Indonesia. All chemical reagents were of analytical grade and used as received. Disodium phosphate (Na_2HPO_4), anhydrous calcium chloride (CaCl_2), and sodium hydroxide (NaOH) were purchased from Merck.

2.2. Methodology

A laboratory-scale experiment utilizing a Continuous Stirred Tank (CSTR) apparatus was executed for ion extraction from seawater. Feed was maintained in a stirred tank agitated at 100 revolutions per minute (rpm) and near ambient temperature. Operational reagents were sodium hydrogen phosphate (Na_2HPO_4) and calcium chloride (CaCl_2), each employed at 15% (w/w) concentration. Residence times of 10, 20, and 30 minutes were investigated. The ion precipitation process commenced with a 1000 mL aliquot of seawater. This sample was first treated with a sodium hydrogen phosphate (Na_2HPO_4) solution. Following this initial step, a calcium chloride (CaCl_2) solution was added in a controlled manner, whereby the mass of the added reagents was adjusted to yield a $\text{Na}_2\text{HPO}_4/\text{CaCl}_2$ weight/weight (w/w) ratio in the experimental range of 0.4 to 0.8—was introduced and stirred for an additional period of 10 minutes. The pH of the suspension was adjusted to 9 by the slow addition of 2N sodium hydroxide, whereupon the stirred suspension was maintained under pH-controlled conditions for the designated residence times of 10, 20, or 30 minutes, consistent with the experimental design. The residual solids were filtered, and calcium and magnesium concentrations were quantified by Atomic Absorption Spectroscopy (AAS).

Concurrent Potassium (K) and Magnesium (Mg) levels were determined by direct titration, with results validated using flame photometry (FP) or atomic absorption spectroscopy (AAS). FP quantifies K emission at 622 nm and Mg at 285 nm, while AAS measures Calcium (Ca) absorption at 205 nm and Mg at 285 nm. These validation methods are chosen for their superior resolution, which stems from high selectivity and low detection thresholds, ensuring accurate quantification of the target metals. For more details, the procedure can be seen in the flowchart shown in the Figure 1.

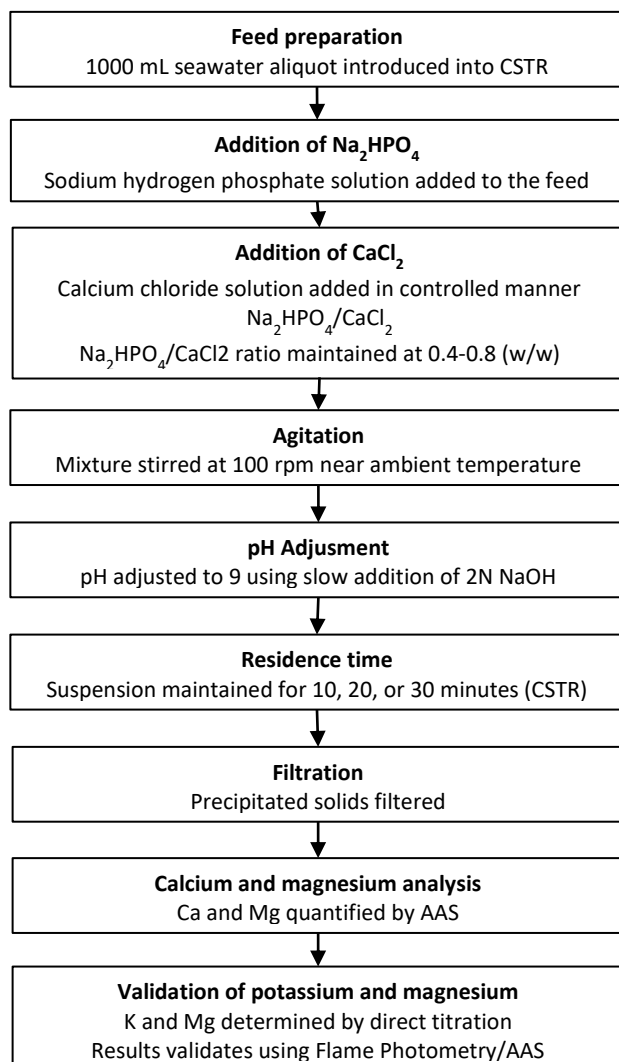


Figure 1. Methodology Flowchart

$$yield = \frac{(C_0 - C_A)}{C_0} \times 100\% \quad (1)$$

Where C_0 is the initial concentration in the untreated seawater feed, while C_A is the final concentration in the treated of solution.

2.3. Response Surface Methodology

Response Surface Methodology (RSM) was utilized to model and optimize the co-precipitation process intended to maximize the simultaneous recovery of magnesium and potassium ions. Experimental design, statistical analysis, and model-fitting were accomplished employing Design-Expert® software (Version 13). Guided by the experimental protocol, two independent factors were selected for investigation: the ratio by volume of sodium hydrogen phosphate to calcium chloride ($\text{Na}_2\text{HPO}_4:\text{CaCl}_2$) and the residence time (minutes). A general central-composite design comprising 11 experimental runs was employed to provide systematic assessment of both the individual and interaction effects of the chosen factors on the two primary responses: magnesium recovery (%) and potassium recovery (%). The final objective of the statistical analysis was to generate an adequate predictive surface and to ascertain the set of operating conditions that yield the maximum recovery efficiency for both magnesium and potassium ions.

The relationship between the independent variables and the response functions was described using a second-order polynomial model, which is the standard form of the response surface in a central composite design. This quadratic model incorporates linear, interaction, and squared terms of the factors, and can be expressed as:

$$Y = \beta_0 + \sum_{i=1}^k \beta_i X_i + \sum_{i=1}^k \beta_{ii} X_i^2 + \sum_{i < j}^k \beta_{ij} X_i X_j + \varepsilon$$

where Y denotes the predicted response, X_i and X_j are the coded independent variables, β_0 is the intercept, β_i represents the coefficients of the linear terms, β_{ii} the quadratic terms, β_{ij} the interaction terms, and ε is the residual error. This model serves as the mathematical basis for evaluating the influence of the process parameters on the recovery of magnesium and potassium, as well as for identifying the optimal operating conditions.

2.4 Face Centered Central Composite Design

A Face Centered Central Composite Design (FC-CCD) was selected for the investigation. Central Composite Designs (CCD) are the prevalent structure within Response Surface Methodology (RSM) and are recognized for their efficiency in constructing a quadratic, or second-order, model without resorting to an exhaustive three-level factorial arrangement. The selected FC-CCD architecture accommodates the simultaneous estimation of linear terms, pairwise interactions, and second-degree effects, thus permitting the characterization of curvature that is vital for the identification of the true global optimum. By encompassing the additional axial and center conditions, the design extends the experimental domain while controlling path-length and design deficiency, ensuring that the constructed model is accurate and generalizes effectively across the entire region of interest. (Variyana et al., 2024)

Table 1. FC-CCD Design Experimental

Run	Ratio Na ₂ HPO ₄ : CaCl ₂	Time (minutes)
1	0.6	20
2	0.6	20
3	0.8	20
4	0.6	10
5	0.4	20
6	0.8	10
7	0.4	10
8	0.4	30
9	0.6	20
10	0.8	30
11	0.6	30

3. Results and Discussion

3.1. Experimental Results

This study focuses on the recovery of magnesium (Mg²⁺) and potassium (K⁺) ions from seawater used in traditional salt production in Sumenep, Indonesia. Table 2 presents the baseline concentrations of the primary cations in the seawater samples. The analysis reveals that magnesium is the most abundant cation, with an average concentration of 861.44 mg/L. In contrast, potassium and calcium (Ca²⁺) are present in considerably lower concentrations, averaging 299.37 mg/L and 273.80 mg/L, respectively. These values establish the initial mineral profile for the subsequent recovery analysis.

Table 2. Seawater analysis

No	Parameter	Average (mg/L)
1	Calcium (Ca)	273.80
2	Magnesium (Mg)	861.44
3	Kalium (K)	299.37

Table 3. magnesium and kalium yield recovery

RATIO Na ₂ HPO ₄ / CaCl ₂	time	Mg ²⁺	K ⁺
		%Yield	%Yield
0.8	10	63.00%	71.50%
	20	92.00%	94.50%
	30	92.70%	96.00%
0.7	10	77.50%	91.00%
	20	93.00%	96.50%
	30	93.00%	96.50%
0.6	10	89.00%	92.50%
	20	97.00%	95.50%
	30	97.00%	95.50%
0.5	10	90.00%	93.00%
	20	97.00%	95.00%
	30	96.00%	95.50%
0.4	10	91.50%	94.00%
	20	97.00%	97.00%
	30	96.00%	97.30%

The data set in the table 3. summarizes the findings of a systematic experiment designed to enhance the quantitative retrieval of magnesium (Mg²⁺) and potassium (K⁺) cations. Examination of the results reveals yield percentage—serving as the central measure of operational effectiveness—to respond markedly to two modulated parameters: the stoichiometric ratio of the dosing salts, Na₂HPO₄ and CaCl₂, and the allotted reaction duration. Widening the experimental time from 10 to 20 minutes produces a pronounced ascendancy in yield for both cations. Simultaneously, efficiency of retrieval exhibits an inverse relationship with the preparative ratio; progressive diminutions of this ratio from 0.8 to 0.4 are seen to produce corresponding raises in yield. Threshold yield is reached when the reaction is conducted with the minimal preparative ratio and the maximal time duration. Under the combinative settings of a 0.4 stoichiometric ratio and a reaction time of 30 minutes, the polychalcogen yield for Mg²⁺ attained 97.00% and for K⁺ 97.30%. Conclusively, the operational recommendations indicated to secure maximal ion sequestration are to maintain a stoichiometric ratio of Na₂HPO₄ to CaCl₂ at or beneath 0.6 and to employ a reaction duration of at least 20 minutes.

3.2. Optimisaztion of precipitation process.

Table 4 summarises the ANOVA for the quadratic model fitted to the magnesium yield data, resolving the influence of the Na₂HPO₄/CaCl₂ ratio (A) and processing time (B). The overall model is confirmed to be statistically adequate ($p=0.0119$), thereby granting confidence to the predictive equations. Main effects are individually significant, with the reactant ratio reporting a p -value of 0.0158 and time a p -value of 0.0093, confirming that both variables independently control yield. The interactive effect represented by the term AB attains significance at $p=0.0299$, emphasising that reaction time modulates the influence of reactant ratio upon yield, necessitating caution in interpreting isolated main effect terms. The Lack of Fit statistic, appearing non-significant ($p=0.0559$), verifies the adequacy of the applying fitted surface and asserts the model's capability to replicate the observed yield with acceptable precision.

Table 4. ANOVA for Quadratic model response yield of Mg^{2+}

Source		Sum Squares	of df	Mean Square	F-value	p-value	
Model		890.33	5	178.07	10.14	0.0119	significant
A-Ratio	$Na_2HPO_4/CaCl_2$	225.71	1	225.71	12.85	0.0158	
B-Time		296.81	1	296.81	16.90	0.0093	
AB		158.76	1	158.76	9.04	0.0299	
A ²		56.46	1	56.46	3.22	0.1329	
B ²		98.04	1	98.04	5.58	0.0645	
Residual		87.81	5	17.56			
Lack of Fit		84.50	3	28.17	17.04	0.0559	not significant
Pure Error		3.31	2	1.65			
Cor Total		978.14	10				

Table 5. Anova for 2FI model response yield of K^+

Source		Sum of Squares	df	Mean Square	F-value	p-value	
Model		385.75	3	128.58	6.59	0.0190	significant
A-Ratio	$Na_2HPO_4/CaCl_2$	115.28	1	115.28	5.91	0.0454	
B-Time		158.11	1	158.11	8.10	0.0248	
AB		112.36	1	112.36	5.76	0.0475	
Residual		136.55	7	19.51			
Lack of Fit		133.39	5	26.68	16.85	0.0570	not significant
Pure Error		3.17	2	1.58			
Cor Total		522.30	10				

The Two-Factor Interaction (2FI) model was subjected to Analysis of Variance (ANOVA) to evaluate the influence of operational variables on product yield (K). The overall model exhibited strong significance, recorded an F-statistic of 6.59, and the associated p-value of 0.0190 reinforces its capacity to accurately forecast the yield. Both main factors were significant: the $Na_2HPO_4/CaCl_2$ ratio (Factor A, $p = 0.0454$) and reaction time (Factor B, $p = 0.0248$). Their interaction (AB) was also significant ($p = 0.0475$), indicating that the effect of reactant ratio depends on reaction time, consistent with prior optimization studies (Montgomery, 2017). Model adequacy was confirmed by a non-significant Lack of Fit ($p = 0.0570$), indicating no systematic deviation from experimental data ((Lee & Yoon, 2021).

The evaluation employed a conventional Analysis of Variance (ANOVA) procedure, applied separately to the recovery datasets for Mg^{2+} and K^+ , and identified a common, statistically significant subset of operational variables. In the surface analyses for both cations, the molar stoichiometric ratio of the reagents (Factor A), the residence time (Factor B), and their paired interaction term (A×B) were confirmed as the principal parameters affecting the recovery yield. A polynomial response surface model developed from these findings is expressed in coded variables, which normalizes the factors to a standard range and thereby permits a straightforward comparison of coefficient magnitudes. Each dimensionless term, whose coefficient magnitudes denote both the strength and direction of influence (Uysal et al., 2019), together with the non-significant Lack of Fit validating model robustness, confirms in accordance with Montgomery, 2017, that the model adequately captures most of the systematic variability in cation recovery.

The distinguishing characteristic between the two-factor interaction (2FI) model and the quadratic model resides in the models' ability to capture and characterize the curvature present in the response surface. The 2FI model represents a first-order approximation, augmented with interaction effects, and retains the postulate that response changes linearly with the experimental factors. Consequently, the model reproduces a hyperplane that can tilt or twist in three-dimensional space but is fundamentally incapable of resolving a local maximum or minimum that is, in reality, constrained to linearity by the model's inherent structure. Conversely, the quadratic formulation introduces squared terms (e.g., factors A and B to the power of 2) and all levels of cross-product terms, transforming the response surface into a second-order polynomial. This enlargement of the parameter space enables the fit of a surface with non-linear, convex or concave curvature, hence permitting the approximation of a maximum crest or a minimum hollow shape, both of which are essential in the context of optimization. Optimization of a surface with such structure mandates a model that can mirror the global shape of the experimental space to a second-order polynomial level, as only these properties allow for the desire to avert the pitfalls

of a misspecified fit. Notable in this regard, extract from Table 4 indicates a non-statistically significant impact of the quadratic terms A^2 and B^2 , implying that the curvature hypothesis, although formally introduced, failed to exhibit sufficient power under the constraints of this specific calibration, and hence the linear terms retain primacy in the model's effective interpretation.

The regression surface in coded variables:

$$\text{Yield (Mg)} = +97.57 - 6.13A + 7.03B + 6.30AB - 4.72A^2 - 6.22B^2 \quad (2)$$

The constant term of +97.57 denotes the anticipated magnesium yield at the centroid of the design matrix. Examination of the linear components reveals that the reaction time variable (B) exerts the dominant positive influence, with a coefficient of +7.03, thus extending the reaction duration enhances yield. Conversely, the $\text{Na}_2\text{HPO}_4/\text{CaCl}_2$ molar ratio (A) demonstrates a pronounced negative coefficient of -6.13, implicating that increasing the ratio constricts magnesium generation. An additional, positive interaction term of +6.30AB suggests that simultaneous augmentation of both independent variables yields a supra-additive effect, offering a greater yield than the isolated additive contributions.

Quadratic terms furnish insights into the curvature of the response surface. The negative coefficient of $-4.72A^2$ and the negative coefficient of $-6.22B^2$ jointly delineate a saddle point, substantiating the presence of a single maximum yield confined to the tested design region. Deviation toward either extremum, excess ratio or extended time, results in diminished magnesium output, thereby necessitating meticulous parameter regulation. The interpretative and anticipative capabilities of the fitted model, therefore, orient subsequent experimental inquiry toward determination of the specific time and ratio values that correspond to the apex magnesium yield, thereby effectively optimizing the recovery process.

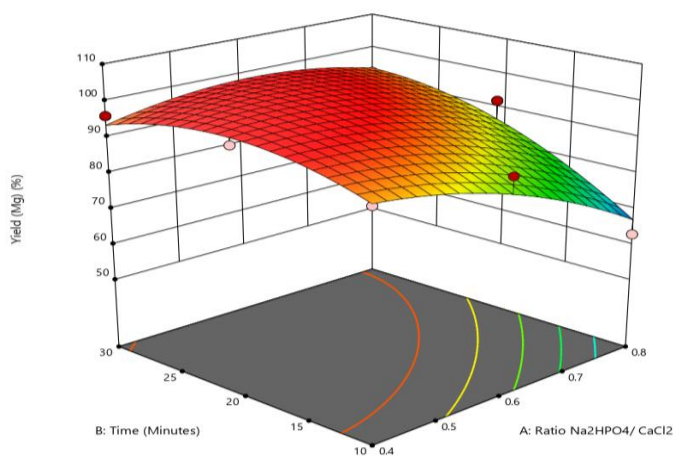


Figure 2. Response surface of yield Mg^{2+}

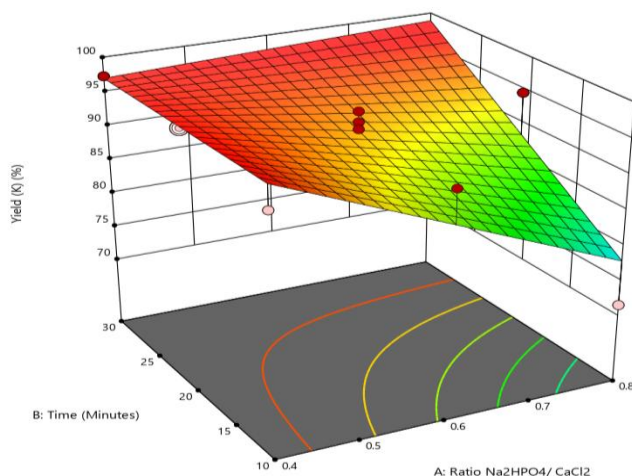


Figure 3. Response surface of yield K^+

Table 6. Optimization of Response Surface Methodology

Number	Ratio Na ₂ HPO ₄ / CaCl ₂	Time	Yield (Mg ²⁺)	Yield (K ⁺)	Desirability
1	0.401	11.608	93.979	97.300	0.908
2	0.400	11.699	94.081	97.300	0.908
3	0.401	11.497	93.854	97.300	0.908
4	0.400	11.881	94.280	97.300	0.908
5	0.400	12.013	94.424	97.300	0.908
6	0.400	12.254	94.677	97.300	0.907

A multi-objective optimization study was applied to refine the chemical manufacture of magnesium and potassium salts by concurrently examining the Na₂HPO₄/CaCl₂ molar ratio, the reaction duration, and the resulting product yields. The preeminent optimal solution specifies an Na₂HPO₄/CaCl₂ ratio of 0.401 and a reaction duration of 11.6 minutes, predicted magnesium and potassium yields attaining 94.0% and 97.3%, respectively. The data, when considered in conjunction with auxiliary trials, demonstrate a systematic pattern by which superior conditions are achieved: a Na₂HPO₄/CaCl₂ ratio constrained to a minimum of 0.4, a reaction duration confined to a tight interval of 11.5 to 12.2 minutes, and a potassium yield approaching its prescribed upper-bound limit. Such parameters delimit a comparatively narrow and reproducible operational envelope that achieves the dual objectives of maximal output and minimized residence time, thereby increasing practical efficiency and process stability.

4. Conclusion

The present investigation has successfully refined the co-precipitation paradigm for the extraction of magnesium and potassium from dilute seawater concentrate, employing Response Surface Methodology. Empirical and statistical analyses identified the Na₂HPO₄/CaCl₂ molar ratio together with the residence time as the paramount perturbations; maximum extraction efficiency was secured at a ratio of 0.401 and a reaction time of 11.6 minutes. These results offer a reliable surrogate model suitable for the scaling of industrial brine valorisation and delineate a systematic pathway for procedural fine-tuning.

Future inquiries should pursue intensified operational efficiency through the integration of emergent deposition accelerative agents such as sonocrystallisation, the modulation of thermal and mechanical disturbance, or a synergistic combination, so as to trim residence exigencies. In parallel, the prospect of employing bioadsorbents emergently warrants systematic appraisal. Pilot-scale evaluation of algal biomass, chitosan derivatives, or post-harvest agrowastes functioning as selective magnesium and potassium carriers remains prudent, for it may proffer a capital-efficient and ecologically benign modality concomitant with circular economy guidelines.

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